Problem sheet 6

Topologie en Meetkunde, Block 3, 2022

April 27, 2022

You do not have to hand-in any solutions to the following exercises. The exercises appear according to topics and, within each topic, in order of difficulty (roughly).

1 Using the universal cover

Exercise 1. Prove that the universal cover of the 2-torus $T^2 := \mathbb{S}^1 \times \mathbb{S}^1$ is $\mathbb{R}^2$. Use this to compute the fundamental group of T^2.

Exercise 2. Compute the fundamental group of the wedge of $\mathbb{S}^1$ and T^2. Can you provide a pictorial description of the universal cover? Prove that it is simply-connected.

Exercise 3. Compute the fundamental group of the wedge of k circles.

Exercise 4. Let A and B be copies of the torus $T^2 := \mathbb{S}^1 \times \mathbb{S}^1$. Compute the fundamental group of

$$C := (A \coprod B)/(A \ni (z,0) \cong (z,0) \in B).$$

2 Constructing covering spaces

Exercise 5. Exercises 14 in page 80 of Hatcher. **Hint:** You should argue geometrically (i.e. you do not need the Galois correspondence). Namely:

- Explain what are the possible local models around different points in $X = \mathbb{RP}^2 \vee \mathbb{RP}^2$. These models will also appear in each covering space Y.

- Produce a cell decomposition of X.

- Y has a cell decomposition that projects down to the one in X. This should allow you to assemble the possible Y.

- The possible ways of attaching the cells will tell you how many possible Y there are.