Timeline of the course

Topologie en Meetkunde, Block 3, 2022

April 27, 2022

In this document you can find the weekly goals for the course (including the topics to be covered and the *minimum* recommended exercises). The deadlines for the homeworks are also included.

In order to pass the course, a student needs to obtain at least a 5 in their final exam. If they do, their final mark will be computed as follows:

- Homeworks 15%.
- Midterm 15%.
- Final exam 70%.

Additionally: Even if their average grade is lower than 5.5, a passing grade for the course will be given if a student obtains a 5.5 or higher in the final exam.

Important comment: In the course we will be using the standard book "Algebraic topology" by Hatcher, as well as my handwritten notes. You should read both, as they approach the material differently. Furthermore, Hatcher covers most of the material of the course but not everything (namely, the material on categories, fundamental groupoid, and surfaces is not there).

1 Week 6. 8th February

Introduction. Homotopy classes of maps. Concatenation of homotopies. Nullhomotopic maps. Contractible spaces. Paths and their concatenation. Path-components.

Hatcher: Pages 1-4 and definition of wedge product in page 10.

Lecture notes: First six pages of "Homotopies of maps".

Exercises: Review sheet of Inleiding Topologie (1,2). Sheet 1 (1,3-5).

Before the lecture: You should watch the introductory video for the course: "A glimpse into Algebraic Topology" https://www.youtube.com/watch?v=61D0sUkqRj8

2 Week 6. 10th February

Categories (objects, morphisms, isomorphisms). The category of topological spaces. The (naive) homotopy category (of topological spaces). Homotopy equivalences as isomorphisms in the homotopy category. Retracts. Deformation retracts.

Lecture notes: "Homotopies of maps". First few pages of "The category of topological spaces".

Exercises: Sheet 1 (9-11,15,17).

3 Week 7. 15th February

Functors. Hom-functors. The pushforward of a map (and its behaviour with retractions and homotopy equivalences). The functor $[X, -]$. Statement of $[\mathbb{S}^1, \mathbb{S}^1] \cong \mathbb{Z}$. Product and coproduct in topological spaces and Sets. The functor $[X, -]$ preserves product and coproduct.

Lecture notes: "More on homotopies". "The category of topological spaces".

Exercises: Sheet 2 (1-4,10,12). You may want to take a look at the sheet on category theory as well.

Before the lecture: You should watch the video "Paths and loops in the real line and the circle" https://www.youtube.com/watch?v=CFtXsm5-JmE

4 Week 7. 17th February

Paths, loops and their concatenation. The fundamental groupoid is a groupoid (the concatenation of paths, up to homotopy, is associative and has identities and inverses). Basic facts about groupoids. The pair groupoid.

Hatcher: Pages 25-27. You may also want to read 21-24 as an introduction to the topic.

Lecture notes: "Fundamental groupoid 1".

Exercises: Sheet 2 (6,7,9).

5 Week 8. 22th February

The category of pointed topological spaces and the fundamental group as a functor. Simply-connected spaces. The fundamental group preserves products. The fundamental groupoid as a functor. The pushforward at the level of fundamental groupoids. Change of basepoint formula.

Hatcher: Pages 28 and 34-37 (but Proposition 1.14 will be proven on week 9, using covering spaces).

Lecture notes: "Fundamental groupoid 2"

Exercises: Sheet 3 (1,6,9,11,12)

Deadline for Homework 1. It must be submitted before the lecture starts (preferably as a PDF produced with latex and through Blackboard).

6 Week 8. 24th February

The category of (pointed) covering spaces. The universal cover. Lifts of maps. Lifts of homotopies. Statement of the homotopy lifting property. Beginning of proof of $\pi_1(\mathbb{S}^1, p) \cong \mathbb{Z}$.

Hatcher: Pages 29-31, 56-57.

Lecture notes: "What are covering spaces?", "Fundamental group of S1".

Exercises: Sheet 3 (10,13,14)

Before the lecture: You should rewatch the video "Paths and loops in the real line and the circle" https://www.youtube.com/watch?v=CFtXsm5-JmE

7 Week 9. 1st March

Wrapping the proof of $\pi_1(\mathbb{S}^1, p) \cong \mathbb{Z}$. Proof of the homotopy lifting property. The higher-dimensional spheres are simply-connected (using covering spaces).

Hatcher: Pages 60-62.

Lecture notes: "Fundamental group of S1". "Homotopy lifting property". "The higher-dimensional spheres are simply-connected".

Exercises: Sheet 4 (1-5).

8 Week 9. 3rd March

Corollaries of $\pi_1(\mathbb{S}^1, p) \cong \mathbb{Z}$: the fundamental theorem of algebra and the Brouwer fixed point theorem. Topologising the fundamental groupoid. The fundamental groupoid as a covering space of the pair groupoid. The universal cover as a subspace of the fundamental groupoid.

Hatcher: Pages 31-34, 63-65.

Lecture notes: "Applications of the fundamental group of the circle", "Fundamental groupoid and universal cover".

Exercises: Sheet 4 (9-11)

Before the lecture: You should watch the video "The fundamental groupoid as a topological space" https://www.youtube.com/watch?v=74tJgCsCZMs

9 Week 10. 8th March

The lifting criterion for maps. Uniqueness of the universal cover. The fundamental group of projective space.

Hatcher: Pages 63-65.

Lecture notes: "Fundamental groupoid and universal cover", "Homotopy lifting property".

Exercises: Sheet 5 (1-4,6).

Deadline for Homework 2. It must be submitted before the lecture starts (preferably as a PDF produced with latex and through Blackboard).

10 Week 10. 10th March

Construction of spaces: Graphs, cell complexes, wedge product. Construction of groups: free groups, group presentations, free product of groups. A first look at the wedge of two circles.

Lecture notes: Beginning of "Cell complexes", and part of "Covering spaces of graphs".

Hatcher: Pages 41-42, 56-60. For more info on cell complexes: 5-14.

Exercises Sheet 5 (13,15,16).

11 Week 11. 15th March

Midterm exam during the lecture hours. It will be closed book.

12 Week 11. 17th March

Using the universal cover to compute π_1. The fundamental group of a wedge of circles/spheres. Some examples of 2-dimensional cell-complexes.

Lecture notes: "Covering spaces of graphs"

Exercises Sheet 6 (1,4,5).

13 Week 12. 22nd March

Some pushout diagrams for groups (amalgamated products of groups) and spaces (unions). Statement of van Kampen. Computation of fundamental groups: higher-dimensional spheres, wedge products, graphs.

Hatcher: Pages 35-36, 49-51, 83-86.

Lecture notes:"Baby van Kampen", "Coproducts", "Theorem of van Kampen".

Exercises: Sheet 7 (4,5,6,12)

Before the lecture: You should watch the video "The theorem of van Kampen" https://www.youtube.com/watch?v=5TBkD1FvlCg

Deadline for Homework 3. It must be submitted before the lecture starts (preferably as a PDF produced with latex and through Blackboard).

14 Week 12. 24th March

Proof of the theorem of van Kampen. The fundamental group of a cell complex.

Hatcher: Pages 40-46.

Lecture notes: "Theorem of van Kampen", "Cell complexes".

Exercises: Sheet 7 (14-17).

15 Week 13. 29th March

Surfaces. Triangulations and planar representations. Structure theorems for surfaces around an edge and a vertex. Invariants: Euler characteristic, orientability, first homology. Statement of the classification theorem. Surgeries: cut and paste, expansion, contraction. Connected sum and the collection of all surfaces as a monoid.

Lecture notes: "Classification of surfaces". "Attaching handles and crosscaps". Take a look at "Simplicial complexes".

Exercises: Sheet 8 (1,2,3,5).

Before the lecture: You should watch the video "Introduction to surfaces" https://www.youtube.com/watch?v=MqdaBsoJF9k

16 Week 13. 31st March

Existence of a planar representation. Proof of the classification of surfaces.

Hatcher: Pages 66-73.

Lecture notes: "Classification of surfaces".

Exercises: Sheet 8 (4,6,7)

Before the lecture: You should watch the video "The classification of path-connected, compact surfaces" https://www.youtube.com/watch?v=2TLu2C-EPe0

17 Week 14. 5th April

The Galois correspondence between pointed covering spaces and subgroups of the fundamental group. Deck transformations.

Lecture notes: "The Galois correspondence"

Exercises: Sheet 9 (1,2,4,8)

Deadline for Homework 4. It must be submitted before the lecture starts (preferably as a PDF produced with latex and through Blackboard).

18 Week 14. 7th April

Review for the exam.